PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 59

Total Marks : 25

Max. Time : 26 min.

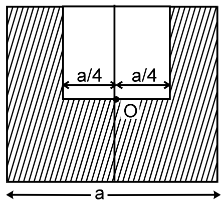
Topics : Rigid Body Dynamics, Circular Motion

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 to Q.4	(3 marks, 3 min.)	[12, 12]
Subjective Questions ('–1' negative marking) Q.5	(4 marks, 5 min.)	[4, 5]
Comprehension ('–1' negative marking) Q.6 to Q.8	(3 marks, 3 min.)	[9, 9]

1. A uniform disc of radius R lies in the x-y plane, with its centre at origin. its moment of inertia about z-axis is equal to its moment of inertia about line y = x + c. The value of c will be

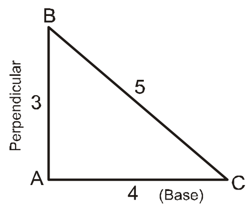
(A) $-\frac{R}{2}$ (B) $\pm\frac{R}{\sqrt{2}}$ (C) $\frac{+R}{4}$ (D) $-R$

2. A square plate of edge a/2 is cut out from a uniform square plate of edge 'a' as shown in figure. The mass of the remaining portion is M. The moment of inertia of the shaded portion about an axis passing through 'O' (centre of the square of side a) and perpendicular to plane of the plate is :



(A) $\frac{9}{64}Ma^2$ (B) $\frac{3}{16}Ma^2$ (C) $\frac{5}{12}Ma^2$ (D) $\frac{Ma^2}{6}$

3. Moment of inertia of uniform triangular plate about axis passing through sides AB, AC, BC are I_P, I_B & I_H respectively & about an axis perpendicular to the plane and passing through point C is I_C. Then :



(A) $I_C > I_P > I_B > I_H$ (B) $I_H > I_B > I_C > I_P$

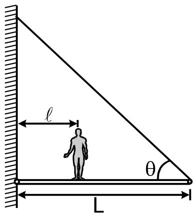
(C) $I_P > I_H > I_B > I_C$ (D) $I_H > I_B = I_C > I_P$

4. Moment of inertia of a uniform quarter disc of radius R and mass M about an axis through its centre of mass and perpendicular to its plane is :

(A) $\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2$ (B) $\frac{MR^2}{2} - M\left(\sqrt{2}\,\frac{4R}{3\pi}\right)^2$

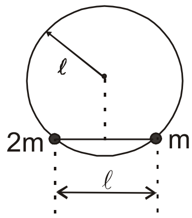
(C) $\frac{MR^2}{2} + M\left(\frac{4R}{3\pi}\right)^2$ (D) $\frac{MR^2}{2} + M\left(\sqrt{2}\,\frac{4R}{3\pi}\right)^2$

5. A uniform horizontal beam of length L and mass M is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle θ with the horizontal. If a man of mass 'm' stands at a distance ℓ from the wall, find the tension in the cable in equilibrium.



COMPREHENSION

Two beads of mass 2m and m, connected by a rod of length ℓ and of negligible mass are free to move in a smooth vertical circular wire frame of radius ℓ as shown. Initially the system is held in horizontal position (Refer figure)



6. The velocity that should be given to mass 2m (when rod is in horizontal position) in counter-clockwise direction so that the rod just becomes vertical is :

(A) $\sqrt{\frac{5g\ell}{3}}$ (B) $\sqrt{\left(\frac{3\sqrt{3}-1}{3}\right)g\ell}$ (C) $\sqrt{\frac{3}{2}g\ell}$ (D) $\sqrt{\frac{5}{2}g\ell}$

7. The minimum velocity that should be given to the mass 2m in clockwise direction to make it vertical is:

(A) $\sqrt{\frac{5g\ell}{3}}$ (B) $\sqrt{\frac{7g\ell}{3}}$ (C) $\sqrt{\left(\frac{3\sqrt{3}+1}{3}\right)g\ell}$ (D) None of these

8. If the rod is replaced by a massless string of length ℓ and the system is released when the string is horizontal then :
(A) Mass 2m will arrive earlier at the bottom.
(B) Mass m will arrive earlier at the bottom.
(C) Both the masses will arrive together but with different speeds.
(D) Both the masses will arrive together with same speeds.

Answers Key

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1. (B) **2.** (B) **3.** (A) **4.** (B)

5. $T = \frac{2mg\ell + MgL}{2L\sin\theta}$ **6.** (B) **7.** (C)

8. (D)

Hint & Solutions

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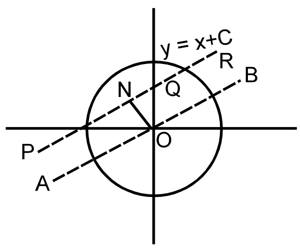
1. $I_{PQR} = I_{AOB} + M.(ON)^2$

$$I_{PQR} = \frac{1}{4}MR^2 + M.\left(\frac{C}{\sqrt{2}}\right)^2$$

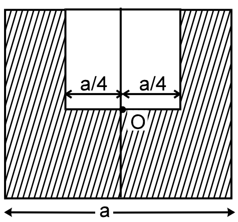
But $I_{PQR} = \frac{1}{2}MR^2$

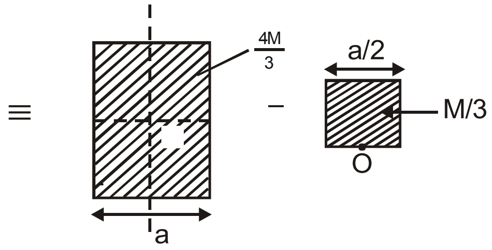


$\therefore \quad C = \pm\frac{R}{\sqrt{2}}$.

Hence (B) is correct.

2.





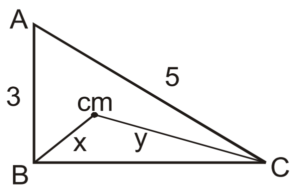
$$I_0 = \frac{(4/3)Ma^2}{6} - \left[\frac{(M/3)(a/2)^2}{6} + \frac{M}{3}\left(\frac{a}{4}\right)^2\right]$$

$$= Ma^2\left(\frac{2}{9} - \frac{1}{72} - \frac{1}{48}\right) = \frac{3Ma^2}{16}$$ **Ans.**

3. Moment of inertia is more when mass is farther from the axis. In case of axis BC, mass distribution is closest to it and in case of axis AB mass distribution is farthest .Hence

$I_{BC} < I_{AC} < I_{AB}$

$\Rightarrow \quad I_P > I_B > I_H$



$I_C = I_{CM} + my^2$

$= I_B^1 - mx^2 + my^2$

$= I_B^1 + m(y^2 - x^2)$

$= I_P + I_B + m(y^2 - x^2)$

$> I_P + I_B$

$> I_P$

Here I_B^1 is moment of inertia of the plate about an axis perpendicular to it and passing through B.

$\therefore \quad I_C > I_P > I_B > I_H$

4. M.I. about 'O' is $\frac{MR^2}{2}$

By parallel-axis theorem : $\frac{MR^2}{2}$

$$= I_{cm} + M\left(\frac{4R}{3\pi}.\sqrt{2}\right)^2$$

$$\Rightarrow I_{cm} = \frac{MR^2}{2} - M\left(\sqrt{2}.\frac{4R}{3\pi}\right)^2$$

5. For rotational equilibrium

Taking torques about A

(so that torque due hinge force on the rod about A = 0)

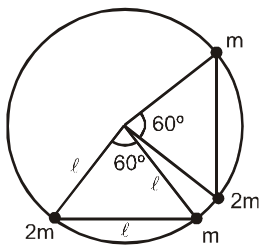
$$mg.\ell + Mg.\frac{L}{2} = T\sin\theta.\ L$$

$$\Rightarrow \mathbf{T} = \frac{2mg\ell + MgL}{2L\sin\theta} \quad \textbf{...Ans.}$$

6. to 8 The speeds given to 2m will also be possessed by m

$\therefore$ KE in horizontal position gets converted in PE in vertical position.

$\frac{1}{2}2mv^2 + \frac{1}{2}mv^2$ = change in PE in vertical position.



Δ PE $= 2\ mg\ [\ell\cos 30° - \ell\cos 60°] + mg\ [\ell\cos 30° + \frac{\ell}{2}]$

$$2\ mg\left[\frac{\ell\sqrt{3}}{2} - \frac{\ell}{2}\right] + mg\left[\frac{\ell\sqrt{3}}{2} + \frac{\ell}{2}\right]$$

$$\therefore \quad mg\ell[\sqrt{3}-1] - mg\ell\left[\frac{\sqrt{3}+1}{2}\right]$$

$$= mg\ell\left[\sqrt{3} - 1 + \frac{\sqrt{3}}{2} + \frac{1}{2}\right] = mg\ell\left[\frac{3\sqrt{3}}{2} - \frac{1}{2}\right]$$

$$K.E. = \frac{1}{2}3mv^2 = mg\ell\left[\frac{3\sqrt{3}-1}{2}\right]$$

$$\therefore v = \sqrt{\left(\frac{3\sqrt{3}-1}{3}\right)g\ell} \quad \textbf{Ans.}$$

7. ||y in anticlockwise direction we get

$$V = \sqrt{\left(\frac{3\sqrt{3}+1}{3}\right) g \ell}$$

8. Both the masses will have same magnitude of acceleration all the time.

$\therefore$ Their velocities and distance covered will be same.

Hence (D).